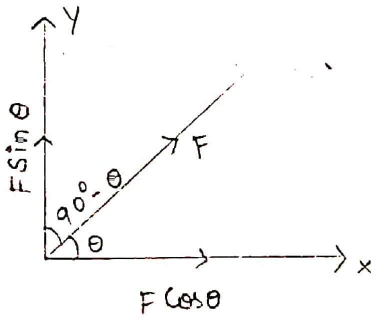
Introduction:

Resolution of a force:

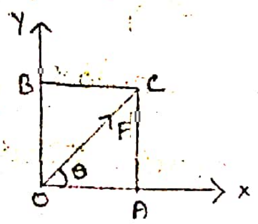
If the force $\vec{F}$ makes an angle θ with the line of direction of ox, then the resolved part of $\vec{F}$ along ox is $F\cos\theta$.

Similarly $F\sin\theta$ is the resolved part of F along oy, the $\perp^r$ direction of ox.



Defn:

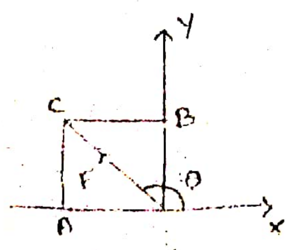
When a given force is resolved into two Components in two mutually $\perp^r$ directions the Components are referred to as the resolved parts in the Corresponding directions.



OA is the resolved part of F along ox

OB is the resolved part of F along oy.

Note:



Here OA is in a direction opposite to ox.

$\Rightarrow$ The resolved part of F along ox is negative.

$\Rightarrow$ $F\cos\theta$ is negative.

Result:

A force F is equivalent to a force $F\cos\theta$ along a line making an angle θ with its own direction together with a force $F\sin\theta$ $\perp r$ to the direction of the first Component.

Corollary: 1

When $\theta = 0$, $\cos\theta = 1$

$\therefore$ The resolued part = F

i.e.) The resolued part of a force in its own direction is the force itself.

Corollary: 2

When $\theta = 90^\circ$, $\cos 90^\circ = 0$

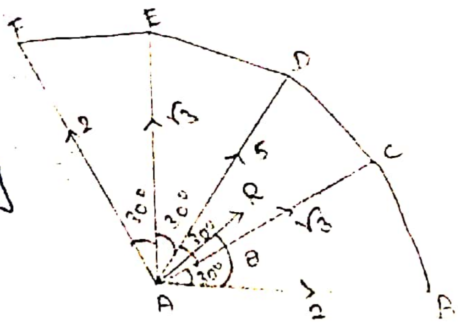
$\therefore$ The resolued part = 0

i.e.) A force has no resolued part in a direction $\perp r$ to itself.

Problems:

a.) Forces of $2, \sqrt{3}, 5, \sqrt{3}, 2$ kgs weight respectively at one of the angular points of a regular hexagon towards the five others in order. Find the direction and magnitude of the resultant.

Soln: Let ABCDEF be a regular hexagon. The forces of $2, \sqrt{3}, 5, \sqrt{3}, 2$ kgs weight at along AB, AC, AD, AE, AF respectively



$\therefore$ Let the magnitude of resultant be R. then

Let R makes angle θ with the resultant R

Resolving along AB direction.

$$R\cos\theta = 2 + \sqrt{3}\cos 30^\circ + 5\cos 60^\circ + \sqrt{3}\cos 90^\circ + 2\cos 120^\circ$$

$$= 2 + \sqrt{3}\cdot\frac{\sqrt{3}}{2} + 5\cdot\frac{1}{2} + \sqrt{3}(0) + 2(-\tfrac{1}{2})$$

$$= 2 + \frac{3}{2} + \frac{5}{2} - 1$$

$$R\cos\theta = 5 \rightarrow ①$$

Resolving along AF direction

$$R\sin\theta = 0 + \sqrt{3}\sin 30^\circ + 5\sin 60^\circ + \sqrt{3}\sin 90^\circ + 2\sin 120^\circ$$

$$= 0 + \sqrt{3}\cdot\frac{1}{2} + 5\cdot\frac{\sqrt{3}}{2} + \sqrt{3} + 2\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2}(1+5+2+2)$$

$$R\sin\theta = 5\sqrt{3} \rightarrow ②$$

$$①^2 + ②^2 \Rightarrow R^2 = 25 + (25\times 3)$$

$$= 25 + 75$$

$$R^2 = 100$$

$$R = 10 \text{ Kgs weight}$$

To find θ:

$$\frac{②}{①} \Rightarrow \tan\theta = \frac{5\sqrt{3}}{5} = \sqrt{3}$$

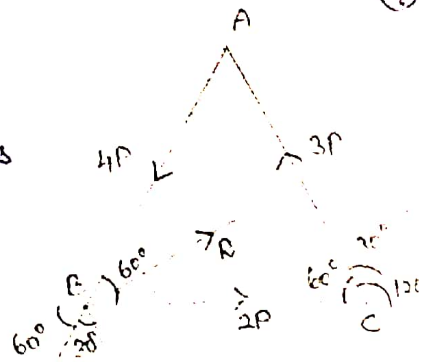
$$\theta = \tan^{-1}\sqrt{3}$$

$$\theta = 60^\circ$$

$\therefore$ magnitude of resultant $R = \cancel{5\sqrt{3}}\ 10$

$\therefore$ direction of resultant $R = 60^\circ$

Q2) The forces 2P, 3P, 4P acting at a point are given by the forces of an equilateral triangle. Find R and θ. (9)



Soln:

Let the 3 forces act on the sides BC, CA, AB of an equilateral triangle

R - Resultant magnitude

θ - R makes angle with BC,

Resolving the forces along BC,

$$2P + 3P\cos 120° - 4P\cos 60° = R\cos\theta$$

$$R\cos\theta = 2P - 3P\left(\tfrac{1}{2}\right) - 4P\left(\tfrac{1}{2}\right)$$

$$= 2P - \frac{3P}{2} - 2P$$

$$R\cos\theta = -\frac{3P}{2} \rightarrow (1)$$

Resolving the forces along direction ⊥r to BC

$$R\sin\theta = 0 + 3P\sin 120° - 4P\sin 60°$$

$$= 3P\cdot\frac{\sqrt{3}}{2} - 4P\cdot\frac{\sqrt{3}}{2}$$

$$R\sin\theta = -\frac{P\sqrt{3}}{2} \rightarrow (2)$$

To find R:

$$(1)^2 + (2)^2 \Rightarrow R^2 = \frac{9P^2}{4} + \frac{3P^2}{4} = \frac{12P^2}{4}$$

$$R^2 = 3P^2$$

$$R = \sqrt{3}\,P$$

To find θ:

$$\frac{(2)}{(1)} \Rightarrow \tan\theta = \frac{-P\sqrt{3}/2}{-3P/2} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \tan\pi/6 = \tan(\pi - 5\pi/6)$$

$$\tan\theta = \tan(\pi - 5\pi/6)$$

$$\theta = 5\pi/6$$

(4) In a square ABCD the forces of magnitude 5P, P, P, 3P act along the sides AB, BC, CD, AD, find the magnitude and direction of the resultant force (92)

Soln:

Resolving along AB,

$R\cos\theta = 5P + 0 - P + 0$

$R\cos\theta = 4P \rightarrow (1)$

Resolving along $\perp$r to AB,

$R\sin\theta = 0 + P + 0 + 3P$

$R\sin\theta = 4P \rightarrow (2)$

$(1)^2 + (2)^2 \Rightarrow R^2 = 16P^2 + 16P^2$

$$\boxed{R = 4\sqrt{2}\,P}$$

$\frac{(2)}{(1)} \Rightarrow \tan\theta = 1 = \tan \pi/4$

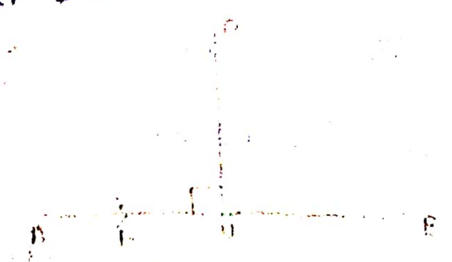
$$\boxed{\theta = \pi/4}$$

Moment of a forces:

Defn:

The moment of a force about a point is defined to be the product of the force and the $\perp$r distance of the point from the line of action of the force.

Let $\vec{F}$ be a force and A is a point on its line of action. Let O be a point in space. Then, the moment of $\vec{F}$ about O $= \vec{OA} \times \vec{F}$.



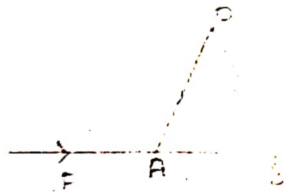
Note: 1

∴ The moment is IP of the position of A on the straight line.

If B is any other point on the line,

then $\vec{OB} \times \vec{F} = (\vec{OA} + \vec{AB}) \times \vec{F}$

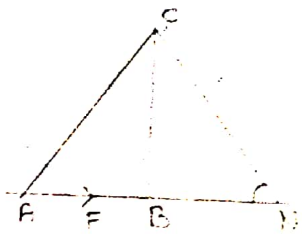
$= \vec{OA} \times \vec{F} + \bar{0}$



Note: 2

If moment of $\vec{F}$ about $A = 0$, then either $\bar{F} = 0$ (or $\bar{OA} = 0$

ie) The line of action of $\vec{F}$ passes through O.



Note: 3

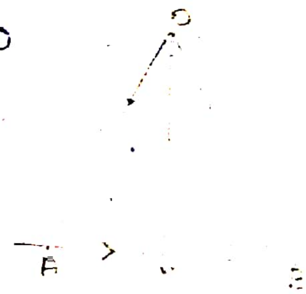
Geometrical Representation of a limit

The moment of the force F about O

$= F \cdot ON$

$= AB \times ON$

$= 2 \Delta AOB$



Hence if a forces is represented by a straight line its moment about any point is given by twice the area of the triangle which the straight line subtends at the point.

Note: Sign of the moment

If the force tends to turn the body in the

(i) anticlockwise direction, the moment is said to be positive

(ii) clockwise direction, the moment is said to be Negative

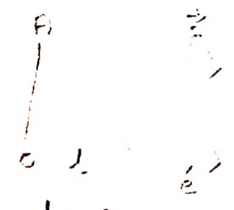
Note:

Thus moment of a force about a point has both magnitude and direction

⇒ It is a Vector quantity.

Moment of a force about a line.

Let $\vec{F}$ be a force



A → a point on the line of action

l → directed line through a point o

$\hat{e}$ → the direction of the line being specified

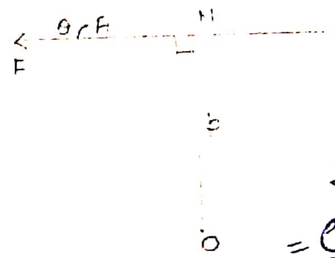
Then, the moment of the force $\vec{F}$ about l

$$= (\bar{OA} \times \bar{F}) \cdot \hat{e}$$

which is the scalar triple product.

Scalar moment:

Let $\vec{F}$ be a force in a plane Let A be a point on its line of action & o, any point in the plane.



$\sin\theta = \frac{ON}{OA}$

ON → ⊥r from o to the line

ON = p

Then the moment of $\vec{F}$ about o is

$$\bar{OA} \times \bar{F} = OA \cdot F \sin\theta \cdot \hat{n} = OA \cdot F \cdot \frac{ON}{OA} \cdot \hat{n}$$

$$= F \cdot ON\, \hat{n}.$$

$$= PF\hat{n}$$

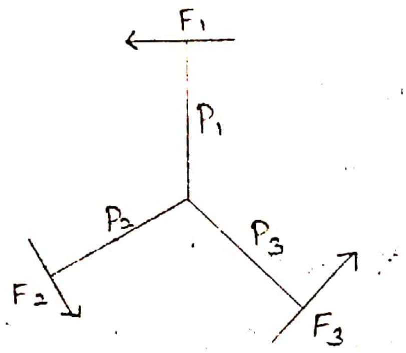
θ → angle b/w OA and $\vec{F}$

$\hat{n}$ → unit Vector ⊥r to OA·$\vec{F}$

The PF is called the scalar moment of $\vec{F}$ about O.

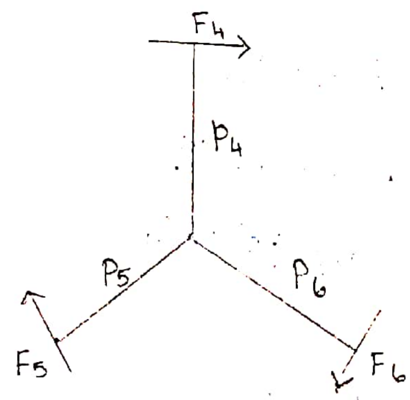
Note:

The scalar moments of F_1, F_2, F_3 are P_1F_1, P_2F_2, P_3 which are positive. These forces Cause on a rigid body a rotational motoin in the anticlockwise



Note:

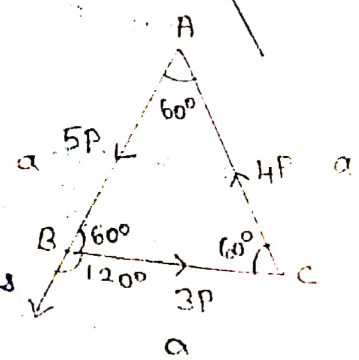
Moment of F_4, F_5, F_6 are P_4F_4, P_5F_5, P_6F_6 which are negative. These three forces Cause a rotational motion in the Clockwise Sense.



a4) Forces of magnitudes 3P, 4P, 5P act along the sides BC, CA, AB of an equileteral triangle of side 'a'. Find the moment of the resultant about A.

Soln:

The moment of the resultant about A $\Big\}$ = $\Bigg\{$ The Sum of the moments of the individual forces about A



$= \vec{AC} \times 3P \cdot \hat{BC}$

The other two forces pass through A.

$\therefore$ moments about A = 0

$= 3P \cdot AC \cdot \hat{AC} \times \hat{BC}$

$= 3P \cdot a \cdot \sin 60^\circ \cdot \hat{k}$ where k is unit ⊥r vector

$= 3P \cdot a \cdot \frac{\sqrt{3}}{2} \hat{k}$

$= \frac{3\sqrt{3}}{2} aP \cdot \hat{k}$

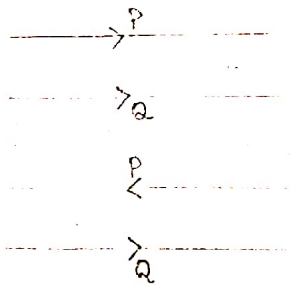
$\therefore$ The moment of resultant forces about A

$= \frac{3\sqrt{3}}{2} aP\hat{k}$.

Parallel forces:

Defn:

Two parallel forces are said to be like when the act in the same direction they are said to be unlike when they act in opposite parallel directions.



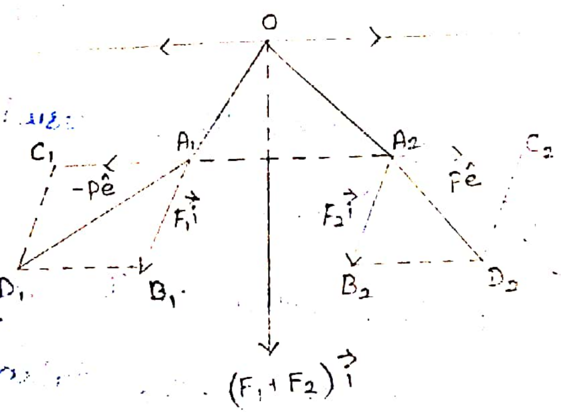
Book Work:

To find the resultant of two parallel forces acting on a rigid body.

Proof:

Case(i)

Let the forces be like parallel forces, namely $F_1\vec{i}$ & $F_2\vec{i}$ acting at A_1 & A_2 respectively



$\vec{i} \to$ unit vector in the direction of the forces.
$\hat{e} \to$ unit vector in the direction of $\overline{A_1A_2}$

Introduce a force $-p\hat{e}$ at A_1 and a force $p\hat{e}$ at A_2. These two forces are equal in magnitude and opposite direction and act along the same line.

$\Rightarrow$ Their introduction will not affect the effects of the given two forces.

Let

$$\overline{A_1B_1} = F_1\vec{i}, \quad \overline{A_1C_1} = -p\hat{e}$$
$$\overline{A_2B_2} = F_2\vec{i}, \quad \overline{A_2C_2} = p\hat{e}$$

Complete the parallelograms $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$

Then the resultant of the two forces $F_1\vec{i}$ and $-p\hat{e}$ acting at A_1 is $\overline{A_1D_1} = F_1\vec{i} - p\hat{e}$ and the resultant of the forces $F_2\vec{i}$ and $p\hat{e}$ acting at A_2 is

$$\overline{A_2D_2} = F_2\vec{i} + p\hat{e}$$

If the lines A_1D_1 and A_2D_2 intersect at O, then the resultant of these two resultant is

$$\overline{A_1D_1} + \overline{A_2D_2} = (F_1\vec{i} - p\hat{e}) + (F_2\vec{i} + p\hat{e})$$
$$= (F_1\vec{i} + F_2\vec{i})$$
$$= (F_1 + F_2)\vec{i}$$

acting at O. This resultant is parallel to the original forces.

Point of intersection of the resultant with A_1A_2:

from the similar triangles ΔOXA_1, $\Delta A_1B_1D_1$

$$\frac{OX}{XA_1} = \frac{F_1}{P} \rightarrow ①$$

Also from the similar triangles OXA_2, $\Delta A_2B_2D_2$

$$\frac{OX}{XA_2} = \frac{F_2}{P} \rightarrow ②$$

$$\frac{②}{①} \Rightarrow \frac{XA_1}{XA_2} = \frac{F_2}{F_1}$$

ie) The line of action of the resultant divides internally A_1A_2 in the ratio $F_2 : F_1$.

Note :

In problems, result is $F_1 \times XA_1 = F_2 \times XA_2$

Position Vector of x :

Let the p.v's of $A_1A_2 = r_1 r_2$

$\because$ x divides A_1A_2 internally in the ratio $F_2 : F_1$ the

$$\text{p.v of } x = \frac{F_1\bar{r}_1 + F_2\bar{r}_2}{F_1 + F_2}$$

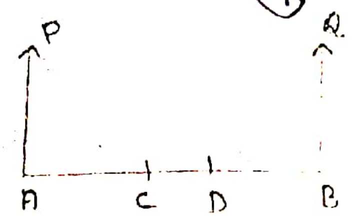
Case (ii)

Let the given forces be unlike parallel forces $F_1\vec{i}$ and $F_2(-\vec{i})$, $(F_1 > F_2)$ acting at A_1 & A_2 respectively

Problems:

b1) Two like parallel forces of magnitude P, Q act on a rigid body. If Q is changed to P^2/Q, with the line of action being the same, S.T the line of action of the resultant will be the same as it would be, if the forces were simply interchanged.

Soln:



Let C be the point in AB where the resultant of the two like parallel forces with P at A & Q at B act.

Then $P AC = Q BC \rightarrow (1)$

If the forces P and $\frac{P^2}{Q}$ act at A, B

[Then the resultant divides AB internally in the ratio $\frac{P^2}{Q} : P \Rightarrow \frac{P}{Q} : 1 \Rightarrow P : Q$]

Let D be the new point of apper of the resultant of the two parallel forces.

then, $P AD = \frac{P^2}{Q} BD$

$PQ AD = P^2 BD$

$Q AD = P BD \rightarrow (2)$

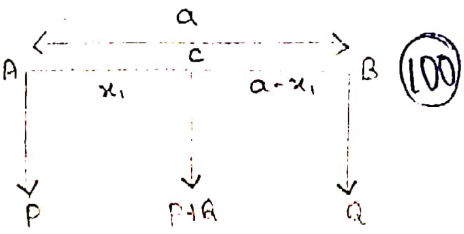
(2) $\Rightarrow$ D is the Centre of 2 like parallel forces with Q at A and P at B.

[For the Second Case also, the ratio is the Same P:Q also all the involved forces & the resultants are $\parallel^{l}$ to one another]

b₂) If two like parallel forces of magnitude P, Q, (P>Q) acting on a rigid body at A, B are interchanged in position. S.T the line of action of the resultant is displaced through a distance. $\frac{AB(P-Q)}{P+Q}$.

Soln:

Let $AB = a$



Let the resultant intersect AB at a distance x_1 from A then

$$x_1 P = (a - x_1) Q$$

$$x_1 P + x_1 Q = aQ$$

$$x_1 (P+Q) = aQ$$

$$x_1 = \frac{aQ}{P+Q} \rightarrow (1)$$

If the distance in the second case is x_2, then

replacing P, Q with Q, P in (1)

$$x_2 = \frac{aP}{P+Q} \rightarrow (2)$$

$$(2) - (1) \Rightarrow x_2 - x_1 = \frac{a}{P+Q}(P-Q)$$

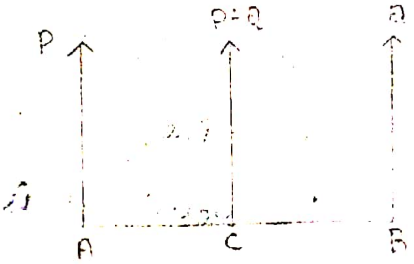
$$= \frac{AB(P-Q)}{P+Q}$$

b3) Two like parallel forces of magnitudes P, Q action a rigid body. If the second force is moved away from the first parallely through a distance d, S.T the resultant of the force moves through a distance $\frac{dQ}{P+Q}$

Soln:

Let P, Q forces act at A, B.

Let their resultant P+Q act at C



then

$$\frac{P}{BC} = \frac{Q}{AC} = \frac{P+Q}{AB}$$

$$\frac{Q}{AC} = \frac{P+Q}{AB}$$

$$AC = \frac{Q}{P+Q} AB \rightarrow (1).$$

When Q is moved parallely through a distance d.
let it act at B'. Now BB' = d.
let the resultant act at c'

Then $\frac{Q}{AC'} = \frac{P}{B'C'} = \frac{P+Q}{AB'}$

$$\Rightarrow Q \cdot AB' = (P+Q)\, AC'$$

$$AC' = \frac{Q \cdot AB'}{P+Q}$$

$$= \frac{Q(AB + BB')}{P+Q}$$

$$AC' = \frac{Q(AB+d)}{P+Q} \rightarrow (2)$$

resultant moves through a distance = cc'

$$AC' - AC = \frac{Q(AB+d)}{P+Q} - \frac{Q \cdot AB}{P+Q}$$

$$= \frac{Q}{P+Q}\left[AB + d - AB\right]$$

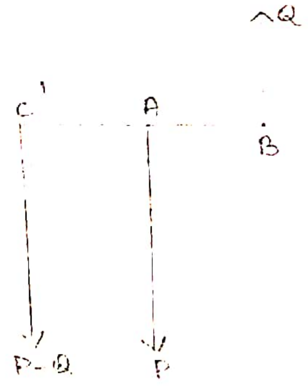
$$\text{distance moved} = \frac{Qd}{P+Q}$$

b4) Two unlike parallel forces P & Q (P > Q) act at A & B respectively s.t if the direction of P be reversed, the resultant is displaced through a distance $\frac{2PQ}{P^2 - Q^2} AB$.

Soln:

Let forces P, Q act at A, B

resultant force = P − Q act at c'

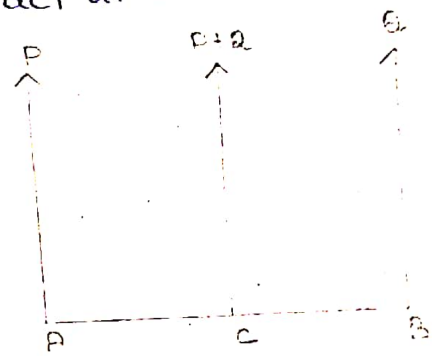


Then $\frac{P}{Bc'} = \frac{Q}{Ac'} = \frac{P-Q}{AB}$

$\Rightarrow \frac{Q}{Ac'} = \frac{P-Q}{AB}$

$Ac' = \frac{AB}{P-Q} \cdot Q \rightarrow (1)$

If P is reversed, let resultant act at c.



Then $\frac{P}{Bc} = \frac{Q}{Ac} = \frac{P+Q}{AB}$

$\Rightarrow \frac{Q}{Ac} = \frac{P+Q}{AB}$

$\Rightarrow Ac = Q \cdot \frac{AB}{P+Q} \rightarrow (2)$

Distance moved by resultant $cc' = Ac + Ac'$

$= Q \frac{AB}{P-Q} + Q \frac{AB}{P+Q}$

$= Q \left[\frac{(P-Q) + (P+Q)}{(P-Q)(P+Q)} \right] AB$

$= \frac{2PQ}{P^2-Q^2} AB$

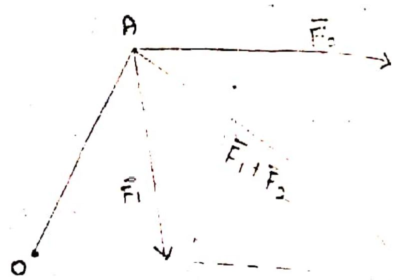
In this Section we bring in the relationship b/w the Sum of the moments of any two coplanar forces and the moment of their resultant.

Varignon's theorem:

The sum of the moments of two intersecting or parallel forces about any point is equal to the moment of the resultant of the forces about the same point.

Proof:

Case (i): Intersecting forces:



Let the lines of action of the forces $\vec{F}_1$ and $\bar{F}_2$ intersect at A. Then the moments of $\bar{F}_1$ and $\bar{F}_2$ about any point O are $\bar{OA}\times\bar{F}_1$, $\bar{OA}\times\bar{F}_2$ and their sum is $\bar{OA}\times\bar{F}_1+\bar{OA}\times\bar{F}_2$.

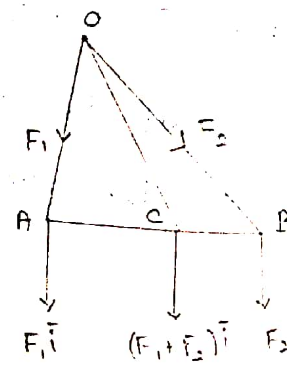
But the resultant of $\bar{F}_1$ and $\bar{F}_2$ is $\bar{F}_1+\bar{F}_2$ acting at A. So its moment about O is

$$\bar{OA}\times(\bar{F}_1+\bar{F}_2)$$

Since $\bar{OA}\times F_1+\bar{OA}\times\bar{F}_2=\bar{OA}\times(\bar{F}_1+\bar{F}_2)$

The theorem follows for the intersecting forces.

Case (ii): Parallel forces:



Let the parallel forces be $\bar{F}_1=F_1\vec{i}$ and $\bar{F}_2=F_2\bar{i}$ acting at A & B. Let $\vec{a}$, $\vec{b}$ be the p.v's of A, B with respect to O. Then the moments of $\bar{F}_1$, $\bar{F}_2$ about O $\bar{a}\times F_1\bar{i}$, $\bar{b}\times F_2\bar{i}$

Their sum is $\bar{a}\times F_1\bar{i}+\bar{b}\times F_2\bar{i}=(F_1\bar{a}+F_2\bar{b})\times\bar{i}$ $\rightarrow(1)$

But the resultant of $F_1\bar{i}$ and $F_2\bar{i}$ is $(F_1+F_2)\bar{i}$ acting at O is

$$\bar{OC} \times (\bar{F_1}+\bar{F_2})\,\bar{i} = \frac{F_1\bar{a}+F_2\bar{b}}{F_1+F_2} \times (F_1+F_2)\bar{i}$$

$$= (F_1\bar{a}+F_2\bar{b}) \times \bar{i} \rightarrow (2)$$

from (1) and (2) we get the theorem for parallel forces.

Remark:

This theorem can be extended easily to any number of coplanar forces.

Let $\bar{F_1}, \bar{F_2}, \ldots \bar{F_n}$ be the given forces.

Then moment of $(\bar{F_1}+\bar{F_2})$ = moment of $\bar{F_1}$ + moment of F_2

Moment of $[(\bar{F_1}+\bar{F_2})+\bar{F_3}]$ = moment of $(\bar{F_1}+\bar{F_2})$ + moment of $\bar{F_3}$

= moment of $\bar{F_1}$ + moment of $\bar{F_2}$ + moment of $\bar{F_3}$

Parallel forces at the vertices of a triangle:

Here we consider the resultant of three like parallel forces acting at the vertices of the triangle.

Problems:

(1) Three like parallel forces P, Q, R act at the vertices of a triangle ABC. S.T their resultant passes through

(i) the Centroid if $P=Q=R$

(ii) the incentre if $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c}$.

Soln:

Let $\bar{a}, \bar{b}, \bar{c}$ be the p.v's of A, B, C. Then the resultant passes through the point whose p.v is 05

$$\frac{P\bar{a} + Q\bar{b} + R\bar{c}}{P+Q+R}.$$

(i) If $P = Q = R$ then

$$\frac{P\bar{a} + Q\bar{b} + R\bar{c}}{P+Q+R} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$$

which is the p.v of the Centroid

(ii) If $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c} = k$ then

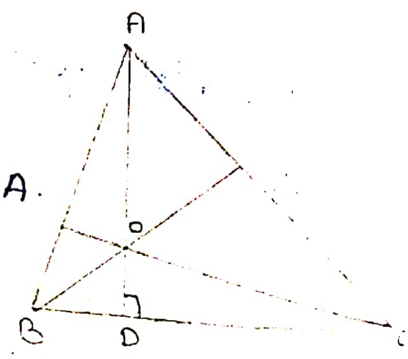
$$\frac{P\bar{a} + Q\bar{b} + R\bar{c}}{P+Q+R} = \frac{k(a\bar{a} + b\bar{b} + c\bar{c})}{k(a+b+c)}$$

$$= \frac{a\bar{a} + b\bar{b} + c\bar{c}}{a+b+c}$$

$\therefore$ which is the p.v of the incentre.

C2) Three like parallel forces P, Q, R act at the Vertices of a triangle ABC. If their resultant passes through the orthocentre O, S.T

$$\frac{P}{\tan A} = \frac{Q}{\tan B} = \frac{R}{\tan C}.$$



Soln:

Let AD be the altitude through A.

Now P acts at A. So that resultant of Q, R should act at D. Such that

$$\frac{BD}{DC} = \frac{R}{Q} \rightarrow (1)$$

But, from $\triangle ABD$, $ABCD$,

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$$BD = \frac{AD}{\tan B}, \quad CD = \frac{AD}{\tan C}.$$

Substituting these Values in ①, we get

$$\frac{Q}{\tan B} = \frac{R}{\tan C}$$

|||ly

$$\frac{P}{\tan A} = \frac{Q}{\tan B}$$

$$\Rightarrow \frac{P}{\tan A} = \frac{Q}{\tan B} = \frac{R}{\tan C}$$

Forces along the sides of a triangle:

In this section we Consider the resultant of forces acting on a rigid body, the forces being along the sides of a triangle.

D,) P, Q, R are forces acting along the sides BC, CA, AB of a triangle ABC taken in order. S.T if their resultant.

(i) Passes through the incentre, then $P+Q+R=0$.

(ii) Passes through the Centroid, then $\frac{P}{a} + \frac{Q}{b} + \frac{R}{c} = 0$ (or)

$$\frac{P}{\sin A} = \frac{Q}{\sin B} = \frac{R}{\sin C}.$$

(iii) Passes through the Circum Centre, then

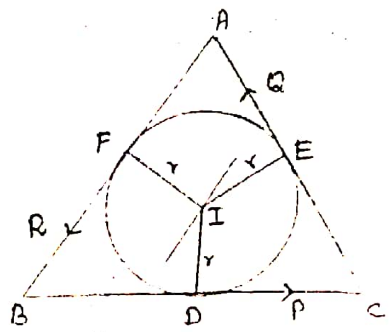
$$P\cos A + Q\cos B + R\cos C = 0.$$

(iv) Passes through the orthocentre, then

$$\frac{P}{\cos A} + \frac{Q}{\cos B} + \frac{R}{\cos C} = 0.$$

Soln:

(i) Incentre I:



Let ABC be the triangle and forces P, Q, R act at the sides BC, CA, AB respectively.

Let I be the incentre. Let ID, IE, IF be the $\perp$r to the sides.

Then $ID = IE = IF = r$

The resultant passes through the incentre and so its moment about I is zero.

$\Rightarrow$ The sum of the moments of the given forces about I is zero.

$$rP + rQ + rR = 0$$

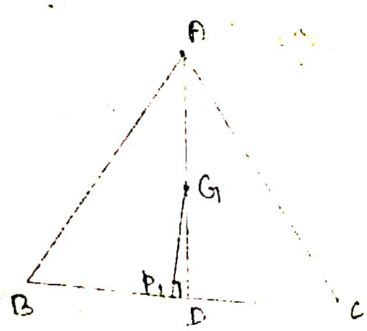
$$\Rightarrow P + Q + R = 0$$

Note:

If P, Q, R are positive, then $P + Q + R \neq 0$ and so the resultant cannot pass through the incentre.

$\therefore$ one or two of P, Q, R must be negative.

(ii) Centroid G:



Let AD be a median. Let p_1 be the length of the $\perp$r from G to BC.

Then the area of ΔABC is

$$\Delta = \frac{1}{2} BC \times (3p_1)$$

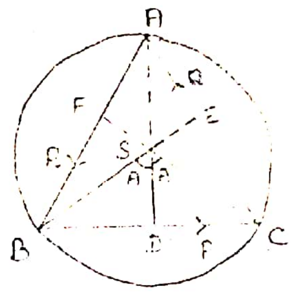
$$\Rightarrow p_1 = \frac{2\Delta}{3a}, \quad p_2 = \frac{2\Delta}{3b}, \quad p_3 = \frac{2\Delta}{3c}.$$

Now, the sum of the moments about G is 0.

$$\therefore\ P\cdot\frac{2\Delta}{3a} + Q\cdot\frac{2\Delta}{3b} + R\cdot\frac{2\Delta}{3c} = 0$$

$$\Rightarrow \frac{P}{a} + \frac{Q}{b} + \frac{R}{c} = 0.$$

(iii) Circum Centre S:



Now A is a point on the circumcircle and So BC Subtends the angle 2A at the Centre S.

SD ⊥r to BC, SE ⊥r to AC,

SF ⊥r to AB

Now $SA = SB = SC = r$ = radius

In ΔBSD,

$$\cos A = \frac{SD}{SB} \Rightarrow SD = r\cos A$$

III^ly $SE = r\cos B$, & $SF = r\cos C$

By Varigon's theorem,

$$P\cdot SD + Q\cdot SE + R\cdot SF = 0$$

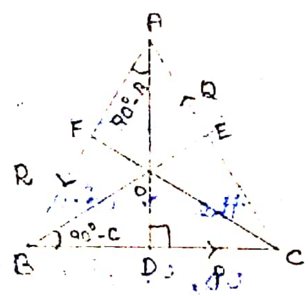
$$Pr\cos A + Qr\cos B + Rr\cos C = 0$$

$$P\cos A + Q\cos B + R\cos C = 0$$

(iv) OrthoCentre O:

AD, BE, CF are three altitudes

O is the orthocentre. Now $\underline{\angle CBE} = 90° - C$



So, from ΔBDO,

$$\frac{OD}{BD} = \tan(90° - C) = \cot C \rightarrow ①$$

But ΔABD

$$\frac{BD}{AB} = \cos B \rightarrow ②$$

① × ②

$$\therefore \frac{OD}{BD} \times \frac{BD}{AB} = \cot C \cos B$$

$$OD = AB \cot C \cos B$$

$$= k \sin C \cdot \frac{\cos C}{\sin C} \cdot \cos B$$

$$OD = k \cos B \cos C$$

$|||^{ly}$

$$OE = k \cos C \cos A$$

$$OF = k \cos A \cos B$$

Taking moments about O, using Varignon's theorem,

$$P \cdot OD + Q \cdot OE + R \cdot OF = 0$$

$$\Rightarrow k(P \cos B \cos C + Q \cos C \cos A + R \cos A \cos B) = 0$$

$\div k \cos A \cos B \cos C \Rightarrow$ $$\frac{P}{\cos A} + \frac{Q}{\cos B} + \frac{R}{\cos C} = 0$$

(or)

$$P \sec A + Q \sec B + R \sec C = 0$$

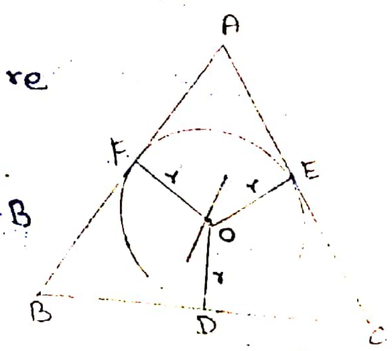
D_2) Three forces P, Q, R act along the sides BC, CA, AB of ΔABC. S.T if their resultant passes respectively through the incentre & circumcentre then

$$\frac{P}{\cos B - \cos C} = \frac{Q}{\cos C - \cos A} = \frac{R}{\cos A - \cos B} \quad \text{(or)}$$

$$P : Q : R = \cos B - \cos C : \cos C - \cos A : \cos A - \cos B.$$

Soln:

Resultant passes through the incentre



Let the radius of the inner circle = r

Let OD, OE, OF be the $\perp$r of BC, CA, AB

$$OD = OE = OF = r = \text{radius}.$$

Since resultant passes through o, moment about o is zero.

By Varignon's theorem,

$$P\cdot OD + Q\cdot OE + R\cdot OF = 0$$

$$Pr + Q\cdot r + R\cdot r = 0$$

$$P + Q + R = 0 \longrightarrow ①$$

Now resultant passes through the Circumcentre.

Let r be the radius of the Circumcircle.

OD, OE, OF are $\perp$r to BC, CA, AB

In $\triangle BOD$

$$\cos A = \frac{OD}{OB} = \frac{OD}{r}$$

$$OD = r\cos A$$

|||ly $\quad OE = r\cos B, \; OF = r\cos C.$

Since resultant passes through o, moment about o is zero.

By Varignon's theorem,

$$P\cdot OD + Q\cdot OE + R\cdot OF = 0$$

$$\Rightarrow P\cdot r\cos A + Q\cdot r\cos B + R\cdot r\cos C = 0$$

$$P\cos A + Q\cos B + R\cos C = 0$$

By using Cross & multiplication rule, solve ① & ②

$$\begin{array}{ccc} P & Q & R \\ 1 & 1 & 1 \\ \cos A & \cos B & \cos C \end{array}$$

$$\therefore \frac{P}{\cos C - \cos B} = \frac{-Q}{\cos C - \cos A} = \frac{R}{\cos B - \cos A}$$

$$\Rightarrow \frac{P}{\cos B-\cos C} = \frac{Q}{\cos C-\cos A} = \frac{R}{\cos A-\cos B} \quad (111)$$

(or) $P:Q:R = \cos B-\cos C : \cos C-\cos A : \cos A-\cos B$

D3) Three forces P, Q, R act along the sides BC, CA, AB of a ΔABC. If their resultant passes through the incentre & Centroid, then S.T. $\frac{P}{a(b-c)} = \frac{Q}{b(c-a)} = \frac{R}{c(a-b)}$

Soln:

The resultant passes through the incentre & Centroid.

$P+Q+R=0 \rightarrow (1)$ refer previous problem

$\frac{P}{a}+\frac{Q}{b}+\frac{R}{c}=0 \rightarrow (2)$ refer problem D, (ii)

Solving (1) & (2)

P	Q	R
1	1	1
$1/a$	$1/b$	$1/c$

$$\frac{P}{1/c-1/b} = \frac{-Q}{1/c-1/a} = \frac{R}{1/b-1/a}$$

$$\frac{Pbc}{b-c} = \frac{-Qac}{a-c} = \frac{Rab}{a-b}$$

$$\frac{Pbc}{b-c} = \frac{Qac}{c-a} = \frac{Rab}{a-b}$$

$\div$ by abc

$$\frac{P}{a(b-c)} = \frac{Q}{b(c-a)} = \frac{R}{c(a-b)}$$

$P:Q:R = a(b-c) : b(c-a) : c(a-b)$

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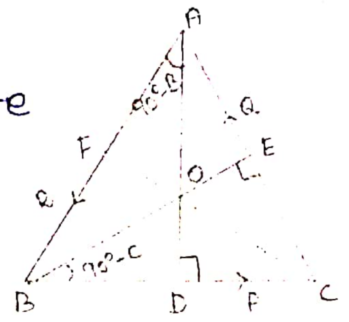
D4) The resultant of three forces P, Q, R acting along the sides BC, CA, AB of a triangle ABC passes through the orthocentre. S.T the triangle must be obtuse angled. If $\angle A = 120^\circ$ and $\angle B = \angle C$. S.T $Q + R = P\sqrt{3}$. (112)



Soln:
Resultant passes through orthocentre

AD, BE, CF are altitudes.

To find $\perp$r distance OD, OE, OF:

In $\triangle BOD$:
$$\frac{OD}{BD} = \tan(90^\circ - C) = \cot C \rightarrow (1)$$

But from $\triangle ABD$,
$$\frac{BD}{AB} = \cos B \rightarrow (2)$$

(1) × (2)
$$\frac{OD}{BD} \times \frac{BD}{AB} = \cot C \cos B$$
$$OD = AB \cot C \cos B$$
$$= k \sin C \frac{\cos C}{\sin C} \cos B$$
$$OD = k \cos C \cos B$$

III^{ly} $OE = k \cos C \cos A$, $OF = k \cos A \cos B$.

Taking moments about O, and using Varignon's theorem
$$P \cdot OD + Q \cdot OE + R \cdot OF = 0$$
$$\rightarrow k(P \cos B \cos C + Q \cos C \cos A + R \cos A \cos B) = 0$$

÷ by $\cos A \cos B \cos C$
$$\frac{P}{\cos A} + \frac{Q}{\cos B} + \frac{R}{\cos C} = 0 \rightarrow (1)$$

magnitude of the forces P, Q, R are

(113)

$\Rightarrow$ Cosine value of it will be obtuse angle

$\Rightarrow$ ABC is obtuse angled triangle

$\angle A = 120^\circ \Rightarrow \cos A = \cos 120^\circ$

$$= \cos(90^\circ + 30^\circ)$$

$$= -\sin 30^\circ$$

$$\cos A = -\frac{1}{2}$$

Also $\angle B = \angle C = 30^\circ$

$\Rightarrow \cos B = \cos C = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$$① \Rightarrow \frac{P}{-1/2} + \frac{Q}{\sqrt{3}/2} + \frac{R}{\sqrt{3}/2} = 0$$

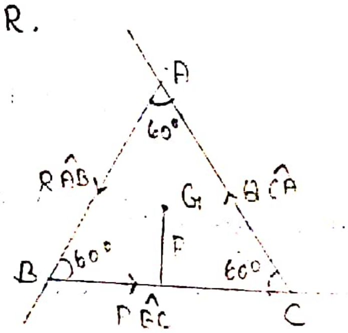
$$P = \frac{Q}{\sqrt{3}} + \frac{R}{\sqrt{3}}$$

$$Q + R = \sqrt{3}\,P$$

D5) Forces $P\hat{BC}$, $Q\hat{CA}$, $R\hat{AB}$ act respectively at B, C, A of an equilateral triangle ABC. If their resultant is a force parallel to BC and through the centroid G of the triangle, S.T $-P = 2Q = 2R$.

Soln:

The resultant force is



$P\hat{BC} + Q\hat{CA} + R\hat{AB}$

This resultant is parallel to BC,

$$\therefore \hat{BC} \times (P\hat{BC} + Q\hat{CA} + R\hat{AB}) = 0$$

$$\Rightarrow 0 + Q\sin 120^\circ - R\sin 120^\circ = 0$$

$$Q = R$$

$\therefore$ The resultant passes through G, the sum of the moments of the force about G = 0. (114)

Thus $P(P+Q+R)=0$

$\Rightarrow P+Q+R=0$

$P+Q+Q=0 \quad [\because Q=R]$

$P=-2Q$

$$\boxed{-P=-2Q=2R}$$